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Optimal design of a variable stiffness joint in a robot manipulator using the response surface method †

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Abstract

The response surface method combined with the design of experiment-based design optimization of a variable stiffness joint (VSJ) is presented in this article. A VSJ used in a manipulator of a robot arm to support 1 kg payload at the end is designed by considering the minimization of the total weight as the objective function. Owing to the requirement of large rotational stiffness of the VSJ, over 10 $N \cdot m$, ring-type permanent magnets are adopted. First, a model composed of two permanent magnets was initially manufactured and tested for comparison with the analysis results. Then, a three-ring-type permanent magnet-based model is suggested and optimized to increase the torque of VSJ. The finite element method is used as a magnetic field analysis method to substitute for the expensive experimental process. Optimization results decrease the weight from 0.899 kg to 0.538 kg, still satisfying the requirement for the rotational stiffness.

Keywords: Variable stiffness joint; Robot manipulator; Permanent magnet; Finite element method; Design of experiments; Response surface method

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1. Introduction

Robots must satisfy the safety requirements as well as their inherent functional requirements so that they can be used in the human living environment to support human beings. Robots interacting with human beings must guarantee safety conditions because they can cause harm to people through abrupt contact. The variable stiffness joint (VSJ) is indispensable to quickly and efficiently eliminate such a kind of shock. Tonietti et al. [1] made a robot arm where the spring and damping property of the linear actuators enabled a variable stiffness. Yun et al. [2] realized a variable stiffness joint concept using rotary-type permanent magnets. When a robot presents a harmful impact on humans, VSJ can rapidly decrease the joint stiffness to reduce the damage as illustrated in Fig. 1.

When a manipulator moves, the VSJ must operate as a joint with proper stiffness so that the robot can perform the expected function exactly and support the required payload. Considering the human living environment, the VSJ must generate at least 10 $N \cdot m$ torsional stiffness to support 1 kg payload at the end of the 1 m-length robot arm manipulator. The size and the weight constraints are also given for the VSJ to be installed in the robot arm joint. It is, however, difficult to design a lightweight and high-performance VSJ using ordinary electric motors because the heavy weight and the large volume are inevitable because of

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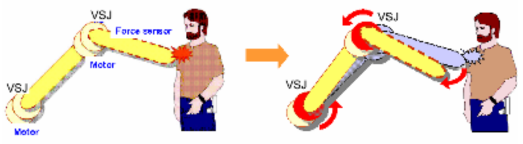


Fig. 1. Performance of VSJ to reduce the impact damage on humans.

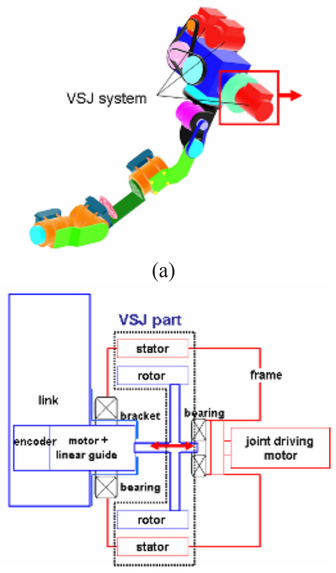
the reduction gear set. In this article, a VSJ using the ring-type neodymium-iron-boron (Nd-Fe-B) permanent magnet (PM) is proposed. The magnet or the magnet applied devices have been used in robot design mostly for the direct current motor used in driving joints such as those in Akadas and Merdrano-Cerda's work for a bipedal robot [3]; however, it is rarely applied to a joint for controlling stiffness in human-interacting situations.

At present, the computational optimization process is indispensable for the electric machinery design process mostly to reduce the weight and improve the performance. Adopting the finite element method (FEM) as a magnetic field analysis method, especially to estimate forces by the Maxwell stress method, an effective optimization approach by the response surface method (RSM), combined with the orthogonal array, is proposed in this article. The design of experiments (DOE) process based on the orthogonal array is performed not only to select the sensitive design variables, but also to determine the 0-level of the following RSM process. The Maxwell stress method shows accurate results although it requires mesh refinement [4, 5]. The RSM has been applied to reluctance motor design [6] or to PM synchronous motor design incorporating genetic algorithm [7]. In this work, the orthogonal array-based DOE is applied to determine sensitive design variables taking into account that their interactions and optimal values of selected design variables are obtained by RSM. This work is based on the simpler three-ring-type VSJ magnetized to the radial direction, while a previous work [8] dealt with the design of another permanent magnet-type VSJ based on the Halbach array concept. The following sections explain in detail the design procedure and the optimization result.

2. Ring-type VSJ using PM

2.1 Subsection VSJ mechanism and torque calculation

In conventional joints in a manipulator, the driving motor gives the rotational stiffness as well. This type of motor is expected to have a reduction gear system for high stiffness ratio; however, it causes disadvantages in terms of space saving, energy saving, and so on; the motor results in rotational stiffness as well. This type of motor is expected to have a reduction gear system for high stiffness ratio; however, it causes disadvantage in space saving, energy saving, and so on. Without adopting the PM, it is difficult to design the VSJ, which is indispensable to the reduction gears.



(a)

(b)

Fig. 2. VSJ application: (a) an example for locations of VSJs in a manipulator; and (b) its mechanism.

Fig. 2 shows the proposed VSJ system composed of the stator and the rotor connected with the joint driving motor. VSJ can be installed in a robot manipulator as displayed in Fig. 2(a). When a robot produces a harmful impact on humans, VSJ can decrease the joint stiffness rapidly to reduce the damage by the mechanism as illustrated in Fig. 2(b). However, when a sensor perceives an impact, it is already late to make the mechanism work to reduce the shock damage; therefore, the mechanism is used mostly to set the stiffness in advance according to the working process. The inner ring (rotor) rotates as the link connected to the manipulator rotates, while the outer ring (stator) is fixed to the frame. In case of the rotor rotation, torque is generated as a result of the restoring force by the misalignment of the magnetic polarization. The rotational stiffness can be altered by moving the inner ring to the axial direction to change the cross-sectional area between the rotor and the stator with the assistance of an additional moving mechanism.

To verify the VSJ performance, it is necessary to compute the torque generated by the relative position between the rotor and the stator. FEM by the commercial package, ANSYS, is used and the virtual work method is applied for the calculation [9]. Although the Maxwell stress tensor method is regarded as a very accurate method, it requires a very fine discretization and it is computationally costly. On the other hand, the virtual work method is less sensitive to the mesh size, although the numerical differentiation of the co-energy is required [5]. This study adopts the virtual work method to avoid the expensive computational cost. The co-energy generated by the magnetic flux must be defined considering the saturation effect.

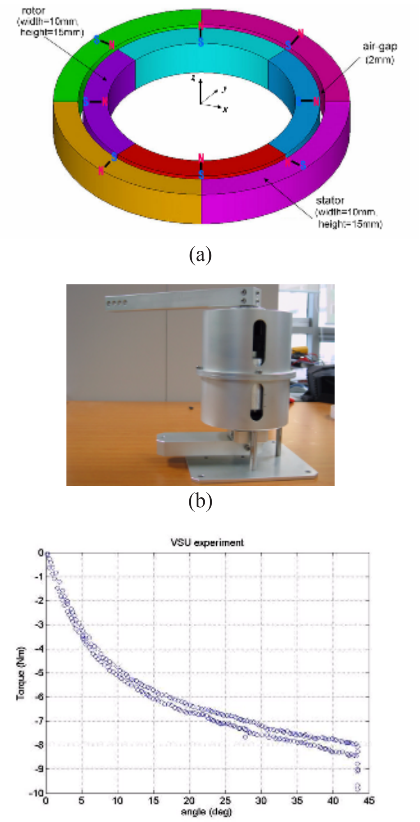
$$W' = \int_V \int_0^B \frac{1}{\mu(B)} B \cdot dB \, dv \tag{1}$$

where W' means energy, and B represents the magnetic flux. V represents the total of the part of interest. $\mu(\mathbf{B})$ is the magnetic permeability considering the saturation effect. Then, the torque can be defined as:

$$T = \frac{\partial W'}{\partial \theta} \tag{2}$$

2.2 Two-ring-type VSJ design and experiment

Fig. 3(a) displays the schematic figure of the two-ring-type VSJ, which is constructed with the inner and the outer magnet rings, and the outer diameter of VSJ is kept as 120 mm. The height of the magnet and the total weight are 15 mm and 0.6835 kg, respectively. The inner ring rotates as the link connected to the robot arm rotates, while the outer ring is fixed to the frame as illustrated in Fig. 2(b). The magnetization direction of the ring magnet is to the radial direction, and it generates torque by the rotation of the inner ring. The variable stiffness may be realized by moving the inner or outer ring to the axial direction to change the cross-sectional area between the rotor and the stator with the assistance of an additional moving mechanism as shown in Fig. 2(b). Using finite element analysis, the maximum torque of the suggested two-ring PM-type VSJ is estimated as over 10 $N \cdot m$. The linear property is maintained at 0 to 10 degrees range of the rotational angle, and it can be used to control the stiffness variation.



(a)

(b)

(c)

Fig. 3. Two-ring PM-type VSJ: (a) schematic figure; (b) experiment set-up; and (c) torque measurement result.

To verify the PM-type VSJ design, an experiment device is set up using the prototype as displayed in Fig. 3(b). Fig. 3(c) shows the torque graph from the experiment results and the maximum torque of the prototype is about 8.4 $N \cdot m$, or 80% of the simulation result although the tendency is similar. The 20% decrease in the maximum torque value may come mostly from the manufacturing or assembling tolerance.

2.3 Three-ring-type VSJ design

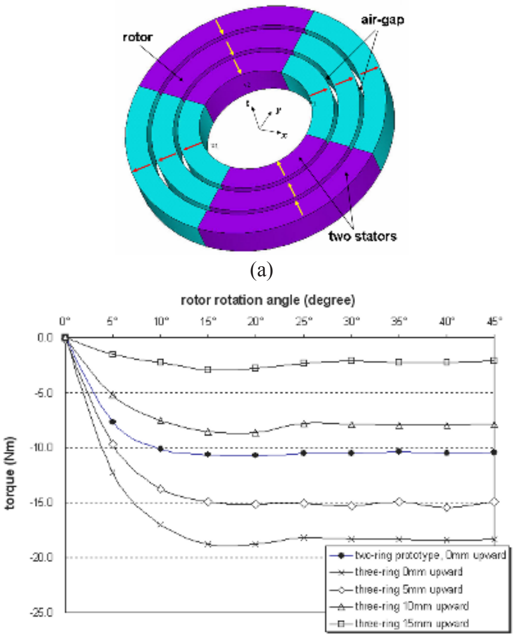
Multiple ring-type PMs are included in the rotor-stator parts to increase the rotational stiffness in a given space limit. Fig. 4(a) shows a potential embodiment of a VSJ composed of three-ring-type PMs, that is, two stators and a rotor. By adding a magnet ring inside the PM rotor, the generated torque is increased owing to the increase of the contact area related with the flux path. The magnet height and air-gaps are set to 15 *mm* and 1 *mm*, respectively, with the width of magnet rings of 10 mm. The air-gap is decreased from 2 *mm* of the previous model (see Fig. 3(a)) to increase the torque value in the possible limit of the fabrication.

By employing the FEM using ANSYS, the analysis results of the generated torque according to the variation of the rotational angle and the rotor movement are given in Fig. 4(b). The torque value becomes more than 15 $N \cdot m$ when the rotor is at the stationary position and it decreases as the rotor part moves up along the *z*-axis as 5, 10, and 15 *mm*, owing to the decrease in the contact area; however, the linear relation between the rotation angle and the torque is still maintained up to the 10-degree rotor rotation angle.

The maximum torque of the three-ring-type VSJ is estimated as 18.4 $N \cdot m$ with 0.899 kg, while that of the two-ring-type VSJ shows 10.8 $N \cdot m$ with 0.683 kg; therefore, the three-ring-type VSJ is selected for the next design process.

3. Design optimization

This article suggests a two-step optimization process composed of the orthogonal array and the RSM. Among design variables, the design of experiments (DOE) the design of experiments (DOE) using the orthogonal array seeks to select a few of them through sensitivity analysis and the verification of interactions between design variables [10]. Design variables affecting the design objective function to the utmost are considered in the optimization routine by the RSM.



(a)

(b)

Fig. 4. Three-ring PM-type: (a) shape showing the magnetization direction; and (b) results of torque analysis according to the variations of the rotational angle and the rotor movement to the axial direction.

3.1 Objective of modification

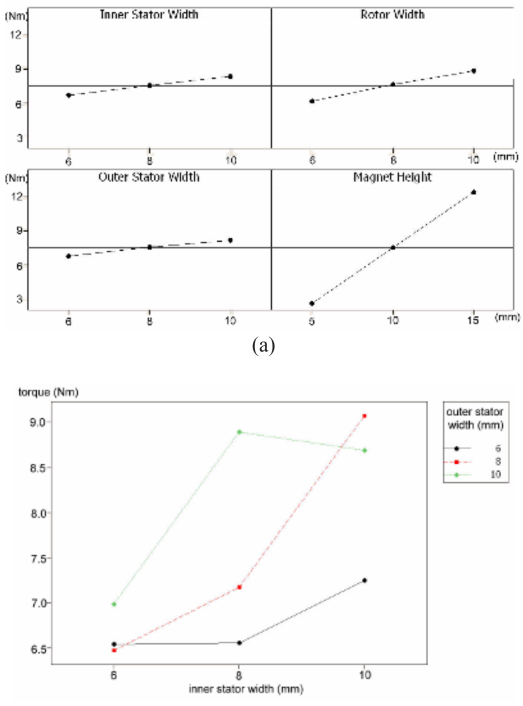
In establishing the objective function, minimizing the total weight is recommended as long as the max-

Table 1. Orthogonal array of DOE with four design variables for VSJ design variables.

Run	Inner stator width (mm)	Rotor width (mm)	Outer stator width (mm)	Magnet height (mm)	Maximum torque (Nm)
1	6	6	6	5	1.7641
2	6	6	8	10	5.5776
3	6	6	10	15	9.7700
4	6	8	6	10	6.2236
5	6	8	8	15	11.1541
6	6	8	10	5	2.5480
7	6	10	6	15	11.6326
8	6	10	8	5	2.6852
9	6	10	10	10	8.6411
10	8	6	6	10	5.6157
11	8	6	8	15	10.0994
12	8	6	10	5	2.3039
13	8	8	6	15	11.3328
14	8	8	8	5	2.6309
15	8	8	10	10	8.2722
16	8	10	6	5	2.7186
17	8	10	8	10	16.0924
18	8	10	10	15	10.1892
19	10	6	6	15	10.1892
20	10	6	8	5	2.3742
21	10	6	10	10	7.3867
22	10	8	6	5	2.6813
23	10	8	8	10	8.4771
24	10	8	10	15	15.2053
25	10	10	6	10	8.8727
26	10	10	8	15	16.3573
27	10	10	10	5	3.4674

Table 2. Experiment array for RSM by central composite method.

Run	Inner stator width (level)	Outer stator width (level)	Magnet height (level)	Maximum torque (Nm)	Weight (kg)
1	-1	-1	-1	2.4556	0.199365
2	1	-1	-1	3.0152	0.245837
3	-1	1	-1	2.8898	0.249555
4	1	1	-1	3.4823	0.299744
5	-1	-1	1	11.7858	0.598095
6	1	-1	1	14.9501	0.737511
7	-1	1	1	14.5473	0.748664
8	1	1	1	17.9126	0.899233
9	-1.68179	0	0	7.1994	0.416819
10	1.68179	0	0	10.3909	0.579211
11	0	-1.68179	0	7.3227	0.410573
12	0	1.68179	0	9.9606	0.585456
13	0	0	-1.68179	0.3857	0.078767
14	0	0	1.68179	19.0163	0.912016
15	0	0	0	8.8609	0.495392



(a)

(b)

Fig. 5. Sensitivity analysis for three-ring PM-type VSJ: (a) analysis results; and (b) interaction between the inner and the outer stator width.

imum torque requirement is guaranteed to reduce the total weight of the manipulator. Therefore, to minimize weight, torque requirement, and geometric constraints, the optimization problem can be defined as follows:

$$\begin{aligned} &\text{minimize } Weight \\ &\text{subject to } \ maximum\ torque \geq 10N \cdot m \\ &\qquad\qquad outer\ diameter \leq 120mm \\ &\qquad\qquad maximum\ height \leq 30mm \end{aligned} \tag{3}$$

The maximum torque requirement is selected considering the difference between the numerical analysis and the prototype performance discussed in the previous section.

3.2 Parameter optimization

The parameter optimization scheme is applied so that the detail size of the three-ring PM-type VSJ is determined. After selecting the design variables, their sensitivity analysis is done using the orthogonal array-based DOE to sort the more effective design variables. Four design variables, such as widths of the rotor and two stators, and the magnet height are selected. The air-gap is fixed at 1 *mm* considering the manufacturing and assembling tolerance. The following three levels of these variables are used in the first DOE process:

Rotor width : 6 *mm*, 8 *mm*, 10 *mm*
Inner and Outer Stator width : 6 *mm*, 8 *mm*, 10 *mm*
PM height : 5 *mm*, 10 *mm*, 15 *mm*

Twenty-seven sets of four design variables are selected for FEM simulations as shown in Table 1 by setting the rotational angle of the VSJ and the maximum torque as the input and the output, respectively. As can be seen from Fig. 5(a), the PM height is verified as the most sensitive design variable. In addition, severe interactions exist between the inner stator width and the outer stator width as shown in Fig. 5(b), while no interaction occurs between the other combinations.

Based on the sensitivity analysis results, insensitive design variables are excluded and the RSM is applied to find the global optimum in the feasible design domain. The RSM is a mathematical method combined with a statistical technique to evaluate responses, that is, the objective quantity influenced by design variables. It intends to find the relationship between the design variables and their response through statistical

Table 3. Comparison of design variables of three-ring PM-type VSJ.

	Inner stator width (mm)	Outer stator width (mm)	Magnet height (mm)
Original model	10	10	15
Optimal model	6.46	6.11	13.06

Table 4. Comparison of results of three-ring PM-type VSJ.

	Original model	RSM prediction	Optimal model
Maximum torque (Nm)	18.4	10.2167	10.3931
Weight (kg)	0.899	0.5373	0.5377

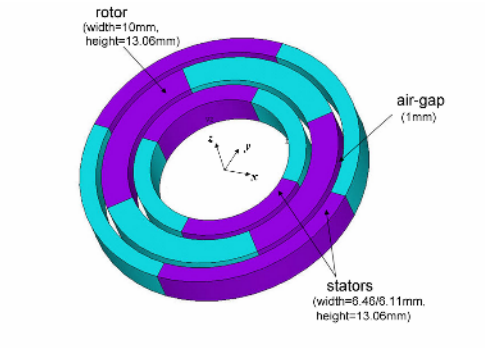


Fig. 6. Optimal shape of three-ring PM-type VSJ.

fitting method. The response is usually obtained from computer simulations or prototype experiments [11]. The purpose of this article is to minimize the total weight of the VSJ with the maximum torque constraint and other geometric constraints. Table 2 shows the experiment array using the central composite design (CCD) with three variables to obtain the regression function by RSM. All the design variables except the rotor width fixed to the optimal level 10 *mm*, are considered in the RSM process mostly to take account of the interaction effect.

Using three design variables and estimated coefficients, the second-order regression model can be represented as follows:

$$y = \alpha_0 + \sum_{i=1}^{3} \alpha_i x_i + \sum_{i=1}^{3} \alpha_{ii} x_i^2 + \sum_{i<j}\sum \alpha_{ij} x_i x_j + \varepsilon \qquad (4)$$

where α_0, α_i, α_{ii}, and α_{ij} are the coefficients; x_1, x_2 and x_3 represent the inner stator width, the outer stator width, and the magnet height, respectively. The regression functions for the maximum torque T and the weight W are defined as follows:

$$\begin{aligned} T = {} & 8.8949 + 0.9555x_1 + 0.8100x_2 + 5.7616x_3 \\ & - 0.0703x_1^2 - 0.1246x_2^2 + 0.2499x_3^2 + 0.0292x_1x_2 \\ & + 0.6722x_1x_3 + 0.6028x_2x_3 \end{aligned} \qquad (5)$$

$$\begin{aligned} W = {} & 0.4954 + 0.0483x_1 + 0.0520x_2 + 0.2483x_3 \\ & + 0.0009x_1^2 + 0.0009x_2^2 + 0.0019x_1x_2 \\ & + 0.0242x_1x_3 + 0.0260x_2x_3 \end{aligned} \qquad (6)$$

The adjusted R^2 value, a basis of precision estimation, is 99.7% in the maximum torque function and 100.0% in the weight function. Table 3 gives a comparison of the design variables for the given design objective, minimizing weight, with constraints. The optimal configuration of the three-ring, PM-type VSJ is shown in Fig. 6.

More results of the optimal model by FEM, as well as the RSM prediction and the results of the original model by FEM, are given in Table 4. It is observed that the FEM results fairly match those of the RSM functions for the optimized model. The weight is reduced to 0.5377 kg, a 40.1% decrease compared to the original model. In spite of the decrease of the maximum torque value, it still has the value of 10.39 $N \cdot m$, satisfying the maximum torque constraint. The maximum torque is somewhat smaller than the result of the previous work using the Halbach array magnetization [8]. However, considering the simple magnetization direction of the current model, it is expected that the suggested design can be competitive from the manufacturing point of view. Although the final design has not been fabricated and tested yet, it is expected that the performance may be acceptable owing to the design margin.

4. Conclusions

This article explains the optimal design process of a VSJ composed of PMs. The PM-type VSJ using two rings is modified to the model with three PM-rings and it is optimized to minimize total weight, while still satisfying the torque and the geometric requirements. The simple size adjustment cannot guarantee the optimal design because of interactive relationships among design variables. Therefore, the optimization technique based on the RSM and the DOE using the orthogonal array is proposed to reduce the total weight of a VSJ, simultaneously considering several design constraints. The result of the optimized VSJ method is confirmed by carrying out the numerical analysis using FEM, thus the proposed method is verified as an appropriate and effective one.

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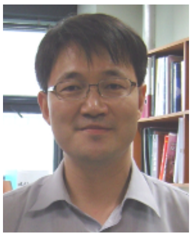

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